Chiral Majorana fermion from quantum anomalous Hall plateau transition


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References

Dirac equation and the anti-particle

In 1928, Dirac unified Einstein’s special theory of relativity with quantum mechanics, and introduced the Dirac equation

\[ i\hbar \gamma^\mu \partial_\mu \psi - mc\psi = 0 \]

where \( \gamma^\mu \) are Dirac’s anticommuting Gamma matrices.

Dirac equation gives negative energy solutions, which led Dirac to predict the existence of anti-particle.

\[ E = \pm \sqrt{p^2 + m^2} \]

In 1932, the positron, the anti-particle of the electron was discovered by CD Anderson in cosmic rays.
Majorana and his fermion

In 1937, Ettore Majorana asked the question: can fermions be their own antiparticles?

The Dirac equation is known to describe charged fermions:

\[ i\hbar \gamma^\mu \partial_\mu \psi - mc\psi = 0 \]

where \( \gamma^\mu \) are Dirac’s anticommuting Gamma matrices.

Majorana claimed if all \( \gamma^\mu \) are selected imaginary, on can make \( \psi \) real, describing a charge neutral, spin \( \frac{1}{2} \) fermion, obeying majorana equation

\[
\begin{align*}
\gamma^0 &= \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, & \gamma^1 &= \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}, & \gamma^2 &= \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \\
\gamma^3 &= \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, & \gamma^5 &= \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix},
\end{align*}
\]

Gamma matrices in Majorana equation.
Properties of the Majorana fermion

Neutrino could be a Majorana fermion, with Majorana mass term.

Majorana fermion is essential for supersymmetry.

Chiral Majorana fermion could exist in 1+1 and 9+1 dimensions, both essential for the superstring theory. With only fermion-number-parity conservation.

Majorana fermion could arise as quasi-particles of topological states of quantum matter.

Majorana fermion could be used for topological quantum computing.

Search for hypothetical particles/wave

Higgs boson, gravitational wave
Majorana fermion
Magnetic monopole
Axion
Dark matter particle
Particle = anti-particle

A Majorana fermion is its own anti-particle

\[ 1 = -1 \]

In other words:

\[ 2 = 0 \]

In the sense of modular (clock) arithmetic like \( 11 + 14 = 25 = 1 \) in clock counting

In particle physics, Majorana neutrino violates lepton number conservation by 2

In condensed matter physics, Majorana fermion are associated with superconductors, which also violates particle number conservation by 2.
Topological insulators and superconductors in 2D

Full pairing gap in the bulk, gapless Majorana edge and surface states

Chiral Majorana fermions $= 1/2$ Chiral fermions

![Diagram showing the relationship between chiral Majorana fermions, chiral SC, QH/QAH, helical SC, QSH, massless Majorana fermions, and massless Dirac fermions.](image)

Qi, Hughes, Raghu and Zhang, PRL, 2009
Basic mechanism of the QAH effect

• Key point to get Quantum Anomalous Hall effect: spin polarized band inversion

Qi, Wu & Zhang, PRB 74, 085308 (2006): general theory
Liu et al, PRL, 101, 146802 (2008): HgMnTe
Yu et al, Science 329, 61, 2010 : \((\text{BiCr})_2\text{Te}_3\)
The model of the 2D topological insulator (BHZ, Science 2006)

Square lattice with 4-orbitals per site:

\[ | s, \uparrow \rangle , | s, \downarrow \rangle , | (p_x + ip_y , \uparrow \rangle , | -(p_x - ip_y) , \downarrow \rangle \]

Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in k.

\[
H_{\text{eff}} (k_x , k_y ) = \begin{pmatrix}
h(k) & 0 \\
0 & h^* (-k)
\end{pmatrix}
\]

\[
h(k) = \begin{pmatrix}
m(k) & A(\sin k_x - i \sin k_y) \\
A(\sin k_x + i \sin k_y) & -m(k)
\end{pmatrix} \equiv d_a(k)\tau^a
\]

\[
\Rightarrow \begin{pmatrix}
m + Bk^2 & A(k_x - ik_y) \\
A(k_x + ik_y) & -m - Bk^2
\end{pmatrix}
\]

\[ m/B < 0, \quad \text{Edge state} \]

\[ m/B > 0, \quad \text{No edge state} \]

Similar to relativistic Dirac equation in 2+1 dimensions, with a mass term tunable by the sample thickness d!
QAH model: FM exchange field

\[ H_{\text{eff}}(k_x,k_y) = \begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix} \quad h_{\text{exchange}}(k) = \begin{pmatrix} \Delta & 0 & 0 & 0 \\ 0 & -\Delta & 0 & 0 \\ 0 & 0 & -\Delta & 0 \\ 0 & 0 & 0 & \Delta \end{pmatrix} \]

\[ h(k) \Rightarrow \begin{pmatrix} m + \Delta + Bk^2 & A(k_x - ik_y) \\ A(k_x + ik_y) & -m - \Delta - Bk^2 \end{pmatrix} \]

\[ h^*(-k) \Rightarrow \begin{pmatrix} m - \Delta + Bk^2 & A(-k_x - ik_y) \\ A(-k_x + ik_y) & -m + \Delta - Bk^2 \end{pmatrix} \]

\[ |m| > |\Delta|, \quad (m + \Delta)/B \quad \text{the same sign} \quad (m + \Delta)/B \]

\[ |m| < |\Delta|, \quad (m + \Delta)/B \quad \text{the opposite sign} \quad (m - \Delta)/B \]
Gapped Dirac fermions on the surface, chiral fermions on the domain wall

- 2D system
- Ferromagnetic
- Topological
- Insulating

QAH can be realized in ferromagnetic TI (Qi, Hughes, Zhang, PRB 2008)
Phase diagram: QAH in a magnetic TI thin film

\[ \mathcal{H}_{\text{surf}}(k_x, k_y) + \mathcal{H}_{\text{Zeeman}}(k_x, k_y) = \begin{pmatrix} 0 & i v_F k_- & m(k) & 0 \\ -i v_F k_+ & 0 & 0 & m(k) \\ m(k) & 0 & 0 & -i v_F k_- \\ 0 & m(k) & i v_F k_+ & 0 \end{pmatrix} + \begin{pmatrix} \Delta & 0 & 0 & 0 \\ 0 & -\Delta & 0 & 0 \\ 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & -\Delta \end{pmatrix} \]

\[ \mathcal{H}_{\text{inv}} = \begin{pmatrix} V & 0 & 0 & 0 \\ 0 & V & 0 & 0 \\ 0 & 0 & -V & 0 \\ 0 & 0 & 0 & -V \end{pmatrix} \]

\[ \Delta^2 > M^2 + V^2 \]

Key message: strong FM ordering

Experimental observation of the QAHE in Cr-BiSbTe3 (Tsinghua 2013, RIKEN, UCLA, Stanford, MIT, Princeton and PSU...)

A

30 mK

\( \rho_{yx}(h/e^2) \)

\( \mu_0 H (T) \)

B

30 mK

\( V_g = -1.5 \ V \)

C

30 mK

\( \rho_{xx}(h/e^2) \)

\( \mu_0 H (T) \)

D

30 mK

\( V_g = -1.5 \ V \)

\( \sigma_{xy}(0), \sigma_{xx}(0) (e^2/h) \)

\( V_g (V) \)
QAH plateau transition: Chalker-Cottington model and effectively tune magnetic exchange coupling

Magnetic TI: Two copy of Dirac model with opposite Chern number

\[ \mathcal{H}_0(k_x, k_y) = \begin{pmatrix} \mathcal{H}_+(k) & 0 \\ 0 & \mathcal{H}_-(k) \end{pmatrix}, \]

\[ \mathcal{H}_\pm(k) = k_y \tau_1 \mp k_x \tau_2 + (m(k) \pm \Delta) \tau_3, \]

\[ C = \begin{cases} \Delta/|\Delta|, & \text{for } |\Delta| > |m_0| \\ 0, & \text{for } |\Delta| < |m_0| \end{cases}. \]

Three kinds of disorder potential:

\[ \mathcal{H}_A = A_x(x,y) \tau_2 \otimes \sigma_3 - A_y(x,y) \tau_1 \otimes 1, \]

\[ \mathcal{H}_\Delta = \Delta(x,y) \tau_3 \otimes \sigma_3, \]

\[ \mathcal{H}_V = V(x,y), \]

Network of chiral edge states at random magnetic domain walls

Critical behavior in QAH plateau transitions
Theoretical Prediction

Two copy of Dirac model with opposite Chern number

1. At coercivity field, and at low enough temperature,

\[ \sigma_{xy} \text{ plateaus at 0!} \]

\[ \left( \frac{\partial \sigma_{xy}}{\partial H} \right)_{\text{max}} \propto T^{-\kappa} \]

\[ \Delta_{1/2} H \propto T^\kappa \]

\[ \kappa \approx 3/14 = 0.214 \]

\[ \kappa = p/2v \]

\[ p \approx 1 \text{ 2D dirty limit} \]

\[ L_{\text{in}}(T) \propto T^{-p/2} \text{ as } T \to 0 \]

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J. Wang, B. Lian, SCZ, PRB 89, 085106 (2014)
Observation of zero Hall plateau state (Tsinghua, UCLA, 2015, RIKEN 2017)
Chiral topological superconductivity in 2D

1 chiral fermion = 2 identical majorana fermion

\[ H_{BdG} = \frac{1}{2} \sum_p \Psi_p^\dagger \begin{pmatrix} h_{QAH}(p) - \mu & i\Delta \sigma^y \\ -i\Delta^* \sigma^y & -h_{QAH}(-p) + \mu \end{pmatrix} \Psi_p \]

\[ h_{QAH}(p) = \begin{pmatrix} m(p) & A(p_x - ip_y) \\ A(p_x + ip_y) & -m(p) \end{pmatrix} \]

X L Qi et al, PRB 82, 184516 (2010)
Model for superconductor proximity coupled QAH effect in magnetic TI

\[ H_{\text{BdG}} = \begin{pmatrix} H_0(k) - \mu & \Delta_k \\ \Delta_k^\dagger & -H_0^*(-k) + \mu \end{pmatrix}, \]

\[ \Delta_k = \begin{pmatrix} i \Delta_1 \sigma_y & 0 \\ 0 & i \Delta_2 \sigma_y \end{pmatrix}. \]

\[ H_0(k) = k_y \sigma_x \tilde{\tau}_z - k_x \sigma_y \tilde{\tau}_z + m(k) \tilde{\tau}_x + \lambda \sigma_z, \]

In a simple case for \( \mu = 0 \) and \( \Delta_1 = -\Delta_2 = \Delta \)

\[ H_{\text{BdG}} = \begin{pmatrix} H_+(k) & 0 \\ 0 & H_-(k) \end{pmatrix}, \]

\[ H_\pm(k) = k_y \sigma_x \mp k_x \sigma_y \sigma_z + [m(k) \pm \lambda] \sigma_z \sigma_\pm \mp \Delta \sigma_y \sigma_y \]

The topological properties of \( H_+ \)

\[ H_+(k) = \begin{pmatrix} h_+(k) & 0 \\ 0 & -h_-^*(-k) \end{pmatrix}, \]

\[ h_\pm(k) = k_y \sigma_x - k_x \sigma_y + [m(k) + \lambda \pm |\Delta|] \sigma_z \]

characterize a px+-ipy superconductor, depending on mass term

Jing Wang et al, PRB 92, 064520 (2015)
Simple criteria: chiral TSC emerges at the QAH plateau transition

Phase boundary:

\[ \Delta \pm (m_0 \pm \lambda) = 0 \]

Usually \( \lambda \gg \Delta, m_0 \)

Competetion between

\[ \Delta \quad m_0 \quad \lambda \]

\[ |m_0| < |\lambda| \quad \rightarrow \quad \text{QAH effect} \]

\[ |\lambda \pm m_0| < |\Delta| \quad \rightarrow \quad \text{Chiral TSC} \]
Phase diagram and realization of TSC in magnetic TI

1. Finite chemical potential.
2. Top and bottom surface better have coupling, otherwise fine tuning of chemical potential into gap is needed.
3. SC proximity only to one surface. (top and bottom have different SC pairing order).

J. Wang et al, PRB 92, 064520 (2015)
Smoking gun of chiral edge Majorana fermion: transport signature

1. QAH and N=2 TSC interface
   - No backscattering
   - Edge current perfectly transmitted

2. QAH and N=1 TSC
   - One chiral majorana complete backscattering
   - The other chiral majorana perfectly transmitted

Smoking gun: transport signature half-integer conductance plateau at the coercive field

½ conductance plateau for N=1 TSC and chiral Majorana edge state

\[ V_1 = -V_2 = \frac{V}{2} \]
\[ I_1 = -I_2 = \frac{e^2}{2h} V \]

J Wang et al, PRB 92, 064520 (2015)
Stability of Chiral TSC against disorder

Quantum percolation theory in D class with particle hole symmetry

Critical scaling: $0 \rightarrow 1/2$, A class
$1/2 \rightarrow 1$, D class

B. Lian, J. Wang, X. Sun, A. Vaezi,
Experimental signature of chiral Majorana edge state

6 quintuple layers
(Cr_{0.12}Bi_{0.26}Sb_{0.62})_2Te_3
2 mm × 1 mm
GaAs (111)B substrate
200 nm layer of Nb
8 mm × 0.6 mm
Experimental signature of chiral Majorana edge state
\( \sigma_{13} \) measurement instead of \( \sigma_{12} \)

**THEORY**

**EXPERIMENT**
Recent comments online from academic community

Journal Club for Condensed Matter Physics

https://www.condmatjclub.org

Mobilizing Majorana fermions

Chiral Majorana fermion modes in a quantum anomalous Hall insulator-superconductor structure
Authors: Qing Lin He, Lei Pan, Alexander L. Stern, Edward C. Burks, Xiaoyu Che, Gen Yin, Jing Wang, Biao Lian, Quan Zhou, Eun Sang Choi, Koichi Murata, Xufeng Kou, Zhijie Chen, Tianxiao Nie, Qiming Shao, Yabin Fan, Shou-Cheng Zhang, Kai Liu, Jing Xia, and Kang L. Wang
Science 357, 294-299 (2017); arXiv:1606.05712

Recommended with a Commentary by Jason Alicea, Caltech
Angels & Demons: Majorana & Dirac fermions in a quantum Hall edge channel

1. A mechanism of $e^2/2h$ conductance plateau without 1D chiral Majorana fermions
   Authors: Wenjie Ji and Xiao-Gang Wen
   arXiv:1708.06214

2. Disorder-induced half-integer quantized conductance plateau in quantum anomalous Hall insulator–superconductor structures
   Authors: Yingyi Huang, F. Setiawan, and Jay D. Sau
   arXiv:1708.06752

Recommended with a Commentary by Carlo Beenakker,
Instituut-Lorentz, Leiden University

Quantum phase transition of chiral Majorana fermion in the presence of disorder,
Summary

1. Chiral topological superconductor can be realized from superconductor proximity coupled quantum anomalous Hall state.

2. Tunable parameters, such as magnetism and hybridization gap make magnetic topological insulator a good platform for chiral TSC.

3. Experimental observation of the $\frac{1}{2}$ plateau as a compelling evidence of chiral Majorana fermion.

4. Hybrid topological materials host a lot of interesting topological phenomena.

Thank you for your attention!