Quantum spin liquids as soft-gap Mott insulators

Yi Zhou (Zhejiang University)

General motivations

- Why quantum spin liquid is interesting?
  - New states of matter in magnetic insulators
  - To understand Mott physics more
Condensed matter physics

- The central issue in condensed matter physics is to discover and understand new states of matters.
Magnetic insulators: Any new states of matter?

- Magnetism is one of the oldest subjects in physics
  - Historically it is associated with magnetic field generated by ordered magnetic moments.
Magnetic insulators: Any new states of matter?

- Magnetic moments order differently in various materials

All these ordered states can be understood from the magnetic interaction between classical vectors (spins).
Magnetic insulators: Any new states of matter?

- What’s the ground state for an antiferromagnet?
- The debate between Néel and Landau
Magnetic insulators: Any new states of matter?

- What’s the ground state for an antiferromagnet?
- The debate between Néel and Landau

- Quantum fluctuations dominate over Néel’s order in one dimension → spin liquid.
- Néel’s order wins at 2D square lattice → AFM order.
- The general situation is still unclear.
Magnetic insulators: **Any new states of matter?**

- It was proposed that quantum spin liquid states can possess very exotic properties.
  - Emerged particles and fields
- **Emergent phenomena**
  - New particles and fields *emerge at low-energy scales but they are totally absent in the Hamiltonian that describes the initial system.*
  - *Different physics laws emerge at different scales.*
Definition: QSL

- Quantum spin liquid (QSL) is an insulator with an odd number of electrons per unit cell which does not order magnetically down to zero temperature due to quantum fluctuations.

  - The quantum disorder is intrinsic, not induced by extrinsic impurities.

  - Not a constructive definition.
Features (or featureless ?)

- “Featureless” Mott insulators.
  - Lattice translational symmetry is respected.
  - The absence of long ranged magnetic order.
  - ... 

- Characterized by emergent phenomena and possible quantum orders
Emergent particles and fields

- **Spinon:** $S=1/2$, charge neutral, mobile objects
  - The spinons may obey Fermi or Bose statistics or even nonabelian statistics and there may or may not be an energy gap;

- **Gauge field:** spin singlet fluctuations
  - These spinons are generally accompanied by gauge fields, $U(1)$ or $Z_2$.

- How can we know which of these plausible states are “real”, especially when $D>1$?
  - *This is a very difficult problem.* I shall outline a few of different approaches to this problem.
Nonlinear $\sigma$-model

- From two-spin dynamics to a rotor model

\[ \vec{S}_i = S\vec{n}_i, \quad |\vec{n}_i| = 1 \quad \Rightarrow \quad \vec{n}_1 = JS^2\vec{n}_2 \times \vec{n}_1, \quad \vec{n}_2 = JS^2\vec{n}_1 \times \vec{n}_2 \]

\[ \vec{L} = \vec{n}_1 + \vec{n}_2, \quad \vec{n} = \vec{n}_1 - \vec{n}_2 \]
\[ \Rightarrow \quad \vec{\dot{L}} = 0, \quad \vec{\dot{n}} = -JS^2\vec{n} \times \vec{L} \]

\[ \vec{r} = r\vec{n}, \quad \vec{L} = \vec{r} \times \vec{p} \Rightarrow \vec{r} \times \vec{L} = -r^2\vec{p} \]
\[ \vec{\dot{r}} = \frac{\vec{p}}{m} = -\left( \frac{1}{mr^2} \right) \vec{r} \times \vec{L}, \quad \vec{\dot{L}} = 0. \]

- Quantum mechanics

\[ \vec{L}^2 = l(l+1)\hbar^2, \quad L_z = m\hbar, \quad m = -l, \ldots, l \]

Ground state: \[ l = 0 \]

Heisenberg uncertainty: \[ \langle \delta \Omega | \delta L \rangle \geq \hbar, \quad \langle \delta L \rangle \rightarrow 0 \quad \Rightarrow \quad \langle \delta \Omega \rangle \rightarrow \infty \]
Nonlinear $\sigma$-model

- Many spin problem: rotor representation

$$H \rightarrow \frac{1}{2I} \sum_i \vec{L}_i^2 + J \sum_{<i,j>} \vec{n}_i \cdot \vec{n}_j$$

$2IJ >> 1 \Rightarrow$ magnetically ordered

$2IJ << 1 \Rightarrow$ spin liquid state

frustration $\Rightarrow$ quantum disordered state

- How does the magnitude of spin enters?

$$S = \int dt \left( \frac{I}{2} \sum_i \dot{\vec{n}}_i^2 + J \sum_{<i,j>} \vec{n}_i \cdot \vec{n}_j \right) + S_{\text{Berry's Phase}}$$

Topological term (F.D.M. Haldane)

Qualitative difference between ground states of integer and half-odd-integer spin chains (Haldane conjugation)
Resonating Valence Bond (RVB)

- Issue: **the nonlinear $\sigma$-model approach becomes too difficult to implement in $D>1$.**

  P.W. Anderson: Why not just “guess” the wave-function...

The term RVB was first coined by Pauling (1949) in the context of metallic materials.
Problem: Is there any simple way to obtain reasonable good variational parameters in RVB wave functions?

Anderson and Zou: We may construct one from BCS wave-function...

\[
|\Psi_{BCS}\rangle = \prod_k \left( u_k + v_k c_k^+ c_{-k}^\downarrow \right) |0\rangle, \quad |\Psi_{RVB}\rangle = P_G |\Psi_{BCS}\rangle
\]

The number of electrons in a BCS wave-function is not fixed. There can be 0, 1, or 2 electrons in a lattice site. The Gutzwiller projection removes all the double occupied components. An insulator state is obtained when # of electron = # of lattice sites.

Later, physicist apply this type of wave-function to different lattice systems with different Hamiltonians and find interesting and rather good results when the \(|\Psi_{BCS}\rangle\) is chosen correctly.
R VB: spinons and gauge fields

• How about excited states?

The spin excitations are electron-like (or superconductor quasiparticle-like, $S=1/2$) except that they do not carry charge, so called spinons.

$$\Psi_{excited} = P_G \gamma_{k\sigma}^+ \gamma_{-k\sigma}^+ \Psi_{BCS}$$

$$\gamma_{k\sigma} = u_k c_{k\sigma} + v_k c_{-k\sigma}$$

**spions:** $S=1/2$, charge neutral, mobile objects

**Question:** Can this simple picture survives Gutzwiller projection?
Confinement of spinons

- X.-G. Wen used the tool of lattice gauge theory to show that spinons may be confined with other spinons to form $S=1$ excitations after Gutzwiller projection.

Structure of projected wave-function

- X.-G. Wen also invented a new mathematical tools (Projective Symmetry Group) to classify spin-liquid states within the Gutzwiller-BCS approach.

Schwinger bosons

- People also use a similar approach where spins are represented by Schwinger bosons. Then spin excitations are $S=1/2$ bosons instead of fermions in projected BCS approach.
Other aspects of theory

- **Exact solvable models**
  - Kitaev honeycomb model changed our view on spin liquids significantly
    - A spin liquid is not necessary to be spin-rotation-invariant.
    - The statistics of spinons can be non-abelian.
    - Spin-orbital effect, etc.
  - ...

- **Other approaches**
  - Analytical methods
    - Bethe Ansatz, Bosonization, CFT, …
  - Numerical methods
    - Exact Diagonalization, DMRG, PEPS / Tensor Network, QMC, CDMFT, …

- **Spin liquids with spin S>1/2**
  - ...

for a review, see YZ, Kazushi Kanoda, Tai-Kai Ng (2014), invited by RMP
Existing $S=1/2$ quantum spin liquid candidates at $D>1$

$k$-(ET)$_2$Cu$_2$(CN)$_3$ (2003)  
$\text{Pd-(dmit)$_2$(EtMe$_3$Sb)}$ (2008)  
$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$ (2007)

$\text{Ba}_3\text{CuSb}_2\text{O}_9$ (2011) quasi 1D?  
$\text{Na}_4\text{Ir}_3\text{O}_8$ (2007)  
$\text{LiZn}_2\text{Mo}_3\text{O}_8$? (2012)
### TABLE III Spin liquid materials summary

<table>
<thead>
<tr>
<th>Material</th>
<th>Triangular $\kappa$-($\text{ET})_2\text{Cu}_2(\text{CN})_3$</th>
<th>Triangular M$[\text{Pd}(\text{dmit})_2]_2$</th>
<th>Kagome $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$</th>
<th>Hyper-Kagome, $\text{Na}_4\text{Ir}_3\text{O}_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific heat</strong></td>
<td>Gapless, $\gamma = 15$ mJ/K$^2$ mol, Field-independent (*2)</td>
<td>Gapless, $\gamma = 20$ mJ/K$^2$ mol, Field-independent (*8)</td>
<td>Gapless, $C \sim T^\alpha$ at low-$T$, $\alpha \leq 1$ (*13), $\alpha = 1.3$ (*14) Field-dependent broad peak (*11, *13)</td>
<td>Gapless, $C \sim T^2$ (*19), $C \sim \gamma T + \beta T^{2.4}$, $\gamma = 2$ mJ/K$^2$ mol (*20), Field-independent (*19, *20)</td>
</tr>
<tr>
<td><strong>Thermal conductivity</strong></td>
<td>Capped; $\Delta = 0.46$ K (*3)</td>
<td>Gapless; finite $\kappa/T$ (*9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>NMR shift</strong></td>
<td>Not precisely resolved (*4)</td>
<td>Not precisely resolved (*10)</td>
<td>A broad peak at 50 K for $^{17}\text{O}$ (*14), at 25-50 K for $^{35}\text{Cl}$ (*15), Finite at low-$T$ (*14)</td>
<td></td>
</tr>
<tr>
<td><strong>NMR $1/T_1$</strong></td>
<td>Inhomogeneous $1/T_1$, Power law, $^1\text{H} 1/T_1; \sim T$ / $\sim T^2$ at $T &lt; 0.3$K (two components) (*1), $^{13}\text{C} 1/T_1; \sim 1/T^{1.5}$ at $T &lt; 0.2$ K (stretched exponential) (*10)</td>
<td>Inhomogeneous $1/T_1$, Power law, $^{13}\text{C} 1/T_1; \sim 1/T^{1.5}$ at $T &lt; 0.2$ K (stretched exponential) (*10)</td>
<td>Power law, $1/T_1 \sim T^\alpha$ at low-$T$, $\alpha \sim 0.73$ for $^{17}\text{O}$ (*14), $\alpha \sim 0.5$ for $^{63}\text{Cu}$ (*15), Field-induced spin freezing (*16)</td>
<td></td>
</tr>
<tr>
<td><strong>$\mu$SR</strong></td>
<td>No internal field at 0 T (*5, *6)</td>
<td></td>
<td>No internal field at 0 T (*17)</td>
<td></td>
</tr>
<tr>
<td><strong>Neutron</strong></td>
<td></td>
<td></td>
<td>Inelastic scattering $\sim$ no excitation gap (*11), Fractionalized excitations with a continuum (*18)</td>
<td></td>
</tr>
</tbody>
</table>
Experimental detection of spin liquid states?

- How to identify and characterize spin liquid states in established materials?

*Patrick A. Lee: All these materials may be described by some kind of projected BCS (or Fermi liquid) state at low temperature.*

P.A. Lee is perhaps the strongest believer of spin liquid states. He works most closely with experimentalist to show that spin liquids exist in nature.

He noticed a common feature of most of the spin liquid candidates discovered so far: they are all close to the metal to insulator transition.

⇒ Question: What is the physical significance of this observation?
Further experimental proposals

- **Optical conductivity**: gapless spinons, power law behavior
  

  Expts: 1) $\kappa$-ET organic salt, S. Elsässer, et. al. (U. Stuttgart group), PRB 86, 155150 (2012).

- **GMR-like setup**: oscillatory coupling between two FM s via a QSL spacer
  

- **Thermal Hall effect**: different responses between magnons and spinons
  

- **Sound attenuation**: spinon-phonon interaction, spinon lifetime, gauge fields
  

- **ARPES**: electron spectral function for a QSL with spinon FS or Dirac cone
  

- **Neutron scattering**: spin chirality, DM interaction, Kagome lattice
  

- **Spinon transport**: measure spin current flow through M-QSL-M junction
  

**Motivation 1**: A generic framework to compare theoretical predictions of QSLs to experimental data is still missing at the phenomenological level.
Power-law dependence of the optical conductivity observed in the quantum spin-liquid compound \( \kappa-(BEDT-TTF)_{2}Cu_{2}(CN)_{3} \)

Sebastian Elsässer, Dan Wu, and Martin Dressel*

Physikalisches Institut, Universität Stuttgart, Pfaffenwaldring 57, D-70550 Stuttgart, Germany

John A. Schlüter
Material Science Division, Argonne National Laboratory, Argonne, Illinois 60439-4831, USA

\[
\sigma(\omega) \sim \omega^n
\]

\[
\eta = \begin{cases} 
3.33, & \omega > \frac{k_B T}{\hbar \tau_{el}} \\
2, & \omega < \frac{k_B T}{\hbar \tau_{el}}
\end{cases}
\]

T.K.Ng and P.A.Lee (2007)

\[1 \text{ cm}^{-1} \sim 3 \times 10^{10} \text{ Hz}\]

\[\hbar \sim 6.6 \times 10^{-16} \text{ eV s}\]

\[1 \text{ cm}^{-1} \sim 2 \times 10^{-5} \text{ eV} \sim 0.2 \text{ K}\]
Difficulties in QSL theory

- What’s the low energy effective theory for QSLs?
  - Confinement vs. deconfinement
  - Spinons are not well defined quasiparticles even though the $U(1)$ gauge field is deconfined. $\Sigma'' \propto \omega^{2/3}$

- Lack of Renormalization-group scheme to illustrate possible effective theories as in Fermi liquid theory.

Motivation 2: Could we construct an effective theory with “electrons” or “dressed electrons” (quasiparticles) directly?

\[ \hat{c}^+ = \hat{f}^+ \hat{h}, \]
\[ \hat{f} \rightarrow e^{i\theta} \hat{f}, \]
\[ \hat{h} \rightarrow e^{i\theta} \hat{h}. \]
Spin liquids in the vicinity of metal-insulator transition

- **Pressure effect**
  - $\kappa-(ET)_2Cu_2(CN)_3$ : $Z_2$ QSL $\rightarrow$ superconductor
  - $Pd-(dmit)_2(EtMe_3Sb)$ : $U(1)$ QSL $\rightarrow$ metal
  - $Na_4Ir_3O_8$ : QSL $\rightarrow$ metal

- **Importance of charge fluctuations**

![Schematic phase diagram on triangular lattice](image)

- Mott transition
- Heisenberg model
- $120^\circ$ AF order
- Fermi liquid
- metal
- insulator
- $U/t$
QSL as a soft-gap Mott insulator

Mott transition

Fermi liquid

metal

insulator

Heisenberg model
AFM order

$U / t$

Schematic DOS (from optical conductivity and other expts.)

$\rho(E) \propto |E - \mu|^{\eta z}$
Questions

• What kind of charge fluctuations should be the key to QSLs?

• Can we formulate a phenomenological theory for QSLs in the vicinity of Mott transition starting from the metallic side?
Quasiparticles

- When electron-electron interactions are adiabatically turned on, the low energy excited states of interacting $N$-electron systems evolve in a continuous way, and therefore remain one-to-one correspondence with the states of noninteracting $N$-electron systems.

- **Assumption**: The same labeling scheme through fermion occupation number can be applied to fermionic QSLs.

Interaction between quasiparticles

\[
\delta E = \sum_{p\sigma} \left( \frac{p^2}{2m^*} - \mu \right) \delta n_{p\sigma} + \frac{1}{2} \sum_{pp'\sigma\sigma'} f_{pp'} \delta n_{p\sigma} \delta n_{p'\sigma'} + O(\delta n^3)
\]

\[
\delta E = E - F_0, \quad \delta n_{p\sigma} = n_{p\sigma} - n_{p\sigma}^0
\]

Quasiparticle energy

\[
\tilde{\varepsilon}_{p\sigma} = \frac{p^2}{2m^*} + \sum_{p'\sigma'} f_{pp'} \delta n_{p'\sigma'}
\]
Landau parameters

Spin symmetric and antisymmetric decomposition

\[ f_{pp'}^{\sigma \sigma'} = f_{pp'}^s \delta_{\sigma \sigma'} + f_{pp'}^a \sigma \sigma' \]

Isotropic systems

3D: \[ f_{pp'}^{s(a)} = \sum_{l=0}^{\infty} f_{l}^{s(a)} P_l(\cos \theta) \]

2D: \[ f_{pp'}^{s(a)} = \sum_{l=0}^{\infty} f_{l}^{s(a)} \cos(l \theta) \]

Dimensionless Landau parameters

\[ F_{l}^{s(a)} = N(0) f_{l}^{s(a)} \]
Ideas

- **Effective theory with chargeful quasiparticles**
  - “Building blocks” are chargeful quasiparticles instead of spinons.
  - Both Fermi liquids and quantum spin liquids can be described within the same framework.

- **Physical quantities will be renormalized**
  - By the interaction between quasiparticles.

- **Electrically insulating but thermally conducting state**
  - Can be achieved in the framework of Landau’s Fermi liquid theory with properly chosen Landau parameters.
Quasi-particle transport (I)

Particle (charge or mass) current carried by quasi-particles

\[ \mathbf{J} = \sum_p \delta \tilde{n}_p \mathbf{v}_p = \sum_p \delta n_p \mathbf{j}_p, \quad \mathbf{j}_p = \mathbf{v}_p - \sum_{p'} f_{pp'} \frac{\partial n^0}{\partial \varepsilon_{p'}} \mathbf{v}_{p'} \]

\[ \delta \tilde{n}_p = n_p - \tilde{n}_p^0 \]

\[ \tilde{n}_p^0 = n_F (\varepsilon_p - \mu) \] is local equilibrium occupation number

⇒ \[ \mathbf{J} = \frac{m}{m^*} \left( 1 + \frac{F^s_i}{d} \right) \mathbf{J}^0 \]

where \( \mathbf{J}^0 \) is the current carried by corresponding non-interacting quasiparticles.
**Quasi-particle transport (I)**

\[
J = \frac{m}{m^*} \left( 1 + \frac{F_{1s}}{d} \right) J^0
\]

*Galilean invariance* is broken by the periodic crystal potential

\[
\Rightarrow \frac{m^*}{m} \neq 1 + \frac{F_{1s}}{d}
\]

charge current is always renormalized by the interaction.

The electron system will be electrically insulating when

\[
1 + \frac{F_{1s}}{d} \rightarrow 0 \quad \text{or} \quad \frac{m}{m^*} \rightarrow 0
\]
Quasi-particle transport (II)

Thermal current carried by quasi-particles

\[ \mathbf{J}_Q = \sum_p \delta \tilde{n}_p (\varepsilon_p - \mu) \mathbf{v}_p = \sum_p (\varepsilon_p - \mu) \mathbf{v}_p \left( \delta n_p - \sum_{p'} \frac{\partial n^0_{p'}}{\partial \varepsilon_p} f_{pp'} \delta n_{p'} \right) \]

\[ \Rightarrow \quad \mathbf{J}_Q = \frac{m}{m^*} \mathbf{J}_Q^0 \]

Thermal current is renormalized by the factor \( m / m^* \)

A quantum spin liquid phase will be achieved through

\[ 1 + \frac{F_{1s}}{d} \to 0, \quad \frac{m}{m^*} \neq 0 \]

strong charge (current) fluctuations in the \( l = 1 \) channel
Framework of effective theory

Effective theory = Landau Fermi-liquid-type theory with chargeful quasiparticles + singular Landau parameters

Elementary excitations (particle-hole excitations) described by Landau transport equation is chargeless at $q=0$ and $\omega=0$, but charges are recovered at finite $q$ and $\omega$. 
Thermodynamic quantities

Specific heat ratio

\[ \gamma = \frac{C_V}{T} = \frac{m^*}{m} \gamma^{(0)} \]

Pauli susceptibility

\[ \chi_P = \frac{m^*}{m} \frac{1}{1 + F_0^a \chi_P^{(0)}} \]

Wilson ratio

\[ R_W = \frac{4\pi^2 k_B^2 \chi_P}{3(g\mu_B)^2 \gamma} = \frac{1}{1 + F_0^a} \]
Electromagnetic responses

Charge response function

\[ \chi_d(q, \omega) = \frac{\chi_{0d}(q, \omega)}{1 - \left( \frac{F_0^s(q, \omega)}{F_1^q(q, \omega)} + d \frac{\omega^2}{v_F^2 q^2} \right) \chi_{0d}(q, \omega) N(0)} \]

Transverse current response function

\[ \chi_t(q, \omega) = \frac{\chi_{0t}(q, \omega)}{1 - \frac{F_1^s(q, \omega)}{F_1^q(q, \omega)} + d \frac{\chi_{0t}(q, \omega)}{N(0)} \chi_{0t}(q, \omega) N(0)} \]

\[ \chi_{0d}, \chi_{0t} : \text{for a Fermi liquid with effective mass } m^* \text{ in the absence of Landau interactions} \]

Longitudinal current response function

\[ \chi_l(q, \omega) = \frac{\omega^2}{q^2} \chi_d(q, \omega) \]

AC conductivity

\[ \sigma_{l(t)}(q, \omega) = \frac{e^2}{i\omega} \chi_{l(t)}(q, \omega) \]

In the limit

\[ 1 + \frac{F_1^s}{d} \to 0 \]

Expand it as

\[ \frac{1 + F_1^s(q, \omega)/d}{N(0)} \sim \alpha + \beta \omega^2 + \gamma_1 q_i^2 + \gamma_i q_i^2 \]

\[ U > U_c \Rightarrow \alpha = 0 \& \gamma_1 = 0 \quad \text{incompressibility} \]

The other possibility \( F_0^s \to \infty \) would result in complete vanishing of charge response.
Dielectric function

\[ \varepsilon(q, \omega) = 1 - \frac{4\pi e^2}{q^2} \chi_d(q, \omega) \sim 1 - 4\pi \beta e^2 v_F^2 N(0)^2 + O(q^2) \]

**Insulator: no screening effect even though we start with the metallic side.**

AC conductivity

For \( q=0 \) and small \( \omega \),

\[ \sigma(\omega) = \frac{\omega \sigma_0(\omega)}{\omega + \left[i/ \beta e^2 N(0)^2 \right] \sigma_0(\omega)} \]

\[ \Rightarrow \quad \text{Re}[\sigma(\omega)] \propto \omega^2 \text{Re}[\sigma_0(\omega)] \]

**Power law AC conductivity inside the Mott gap.**
Quasiparticle scattering amplitude

\[ A_{pp'}(q, \omega) - \sum_{p''} f_{pp''} \chi_{0p''}(q, \omega) A_{p''p'}(q, \omega) = f_{pp'} \]

where

\[ \chi_{0p''}(q, \omega) = \frac{n_{p-q/2}^0 - n_{p+q/2}^0}{\omega + \xi_{p-q/2} - \xi_{p+q/2}} \approx \frac{q \cdot v_p}{\omega} \frac{\partial n_p^0}{q \cdot v_p - \omega \partial \xi_p} \]

Assuming that the scattering is dominating in the \( l = 1 \) channel,

\[ f_{pp'} \sim \frac{p \cdot p'}{P_F^2} f_1^s \]

Using \[ \frac{1 + F_1^s(q, \omega) / d}{N(0)} \sim \beta \omega^2 + \gamma q_i^2 \], after some algebra, we obtain

\[
A_{pp'}(q, \omega) \approx \frac{d}{N(0)} \frac{p \cdot p'}{P_F^2} \frac{1}{\omega - ig \frac{\omega}{v_F q} + \gamma q^2}, \quad g = \begin{cases} 1, & d = 2 \\ \frac{\pi}{2}, & d = 3 \end{cases}
\]
Thermal conductivity

Thermal resistivity for a Fermi liquid

\[
\frac{1}{\kappa} = \frac{1}{4} \sum_{1,2,3,4} W(1,2;3,4) n_1^0 n_2^0 (1 - n_3^0)(1 - n_4^0) \left( \phi_1 + \phi_2 - \phi_3 - \phi_4 \right)^2 \left( \sum_1 \phi_1 \xi_1 \mathbf{v}_1 \cdot \mathbf{u} \frac{\partial n_1^0}{\partial \varepsilon_1} \right)
\]

\[
\times \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \delta_{\sigma_1+\sigma_2,\sigma_3+\sigma_4} \delta_{p_1+p_2,p_3+p_4} \quad \text{C.J. Pethick, Phys. Rev. 177, 393 (1969)}
\]

\[
n_i = n_0 - \phi_i \frac{\partial n_i^0}{\partial \varepsilon_i}, \quad \mathbf{v}_i \text{ is quasiparticle velocity, } \mathbf{u} \text{ is an arbitrary unit vector along } \nabla T
\]

Trial functions: \( \phi_i = \xi_i \mathbf{v}_i \cdot \mathbf{u}, \quad \xi_i = \varepsilon_i - \mu \)

Scattering probability:

\[
W(1,2;3,4) = 2\pi |A_{pp'}(\mathbf{q}, \omega)|^2
\]

\[
\omega = \varepsilon_{p+q/2} - \varepsilon_{p-q/2} = \varepsilon_{p'+q/2} - \varepsilon_{p'-q/2}
\]
Thermal resistivity due to inelastic scattering between quasiparticles

\[
\frac{1}{\kappa_{in}} \propto \left( \frac{k_B T}{\epsilon_F} \right)^{(d-1)/3}
\]

Thermal resistivity due to elastic impurity scattering

At low temperature, the inelastic scattering is cut off by elastic impurity scattering rate \( \frac{1}{\tau_0} \)

\[
\frac{\kappa_{el}}{T} = \frac{1}{d} \gamma^* v_F^2 \tau_0
\]

\[
\Rightarrow \quad \frac{\kappa}{T} \propto \max \left[ \frac{\hbar}{k_B^3} \left( \frac{k_B T}{\epsilon_F} \right)^{(4-d)/3}, \frac{d}{\gamma^* v_F^2 \tau_0} \right]^{-1}
\]

Consistent with U(1) gauge theory,

Collective modes

Charge sector: density fluctuations

\[
\delta n_p = - \frac{\partial n_p^0}{\partial \varepsilon_p} \nu_p \quad \text{energy shift in the direction } \hat{p}
\]

spherical symmetry \[\Rightarrow \quad \nu_p = \sum_l \sum_{m=-l}^l Y_l^m(\theta_p, \phi_p) \nu_l^m\]

Equation of motion: linearized Landau transport equation

\[
\frac{\partial \delta n_p}{\partial t} + \vec{v}_p \cdot \nabla_r \left( \delta n_p - \frac{\partial n_p^0}{\partial \varepsilon_p} \delta \varepsilon_p \right) = I[n_{p'}]
\]

collision integral
Density fluctuation modes in a system with spherical symmetry

- $m = 0$: longitudinal mode
- $m = 1$: transverse mode
- $m = 2$: quadrupolar mode
Zero sound modes in the spin liquid phase:  \[ 1 + \frac{F_s}{3} = 0 \]

Three channel model with only \( F_0^s, F_1^s \) and \( F_2^s \): a weakly damping zero sound mode exists when \( F_2^s > 10/3 \).

\[
S \equiv \frac{\omega}{q v_F}
\]

\[
S = \begin{cases} 
\sqrt{\frac{175}{F_2^s}}, & F_2^s \gg \frac{10}{3}, \\
\frac{20 - 11F_2^s}{1}, & 0 < F_2^s - \frac{10}{3} << 1
\end{cases}
\]
Different from Brinkman-Rice picture
Summary

- **Mott transition driven by current fluctuations**
  - An alternative picture of metal-insulator transitions to Brinkman-Rice
  - QSL as a soft gap Mott insulator

- **Phenomenological theory for both Fermi liquids and quantum spin liquids in the vicinity of Mott transition.**
  - FL: both electrically and thermally conducting
  - QSL: electrically insulating but well thermally conducting

- **Mott physics**: characterized by many intrinsic in-gap excitations in such quantum spin liquids.
  - Wilson ratio ~ 1, ac conductivity, dielectric function, thermal conductivity
  - There exist collective modes as well as “quasiparticles”
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Thank you for attention!