Magnetic Star - Disk Interaction

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Magnetic Star - Disk Interaction: Basic Picture

$rm$ Magnetosphere radius
Magnetic Star - Disk Interaction: Basic Picture

Magnetosphere boundary radius:
For spherical accretion:

\[
\frac{B^2}{8\pi} \sim \frac{1}{2} \rho V^2 \quad \text{with} \quad B \sim \frac{\mu}{r^3}, \quad V \sim \sqrt{\frac{GM}{r}}, \quad \rho = \frac{\dot{M}}{4\pi r^2 V} \quad \Rightarrow \quad r_m \sim \left(\frac{\mu^4}{GMM^2}\right)^{1/7}
\]
Magnetic Star - Disk Interaction: Basic Picture

Accreting x-ray pulsars:
- $B_* \sim 10^{12} \text{G}$,
- $r_m \sim 10^2 R_*$

Accreting ms pulsars:
- $B_* \sim 10^8 \text{G}$,
- $r_m \sim (\text{a few}) R_*$

Accreting WDs (Intermediate polars):
- $B_* \sim 10^7 \text{G}$,
- $r_m \sim 10 R_*$

Protostars (Classical T Tauri stars):
- $B_* \sim 10^3 \text{G}$,
- $r_m \sim (\text{a few}) R_*$

\[ r_m \sim \left( \frac{\mu^4}{GM^2} \right)^{1/7} \]

\[ \frac{B(r)^2}{8\pi} \sim \frac{1}{2} \rho V_r V_\phi \]
Application:
QPOs in Accreting Millisecond Pulsars

Van der Klis 2005
Application:
Spinup/Spindown of Accreting X-ray pulsars

Camero-Arranz et al. 2010, 2012
Application:
Rotation of Protostars: why 10% of breakup?

Gallet & Bouvier 2013
Application:
Outflows/jets from protostars:
Magnetic Star - Disk Interaction: Basic Picture

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Simulations...

Hayashi, Shibata & Matsumoto, Miller & Stone, Goodson, Winglee & Bohm, Fendt & Elastner, Matt et al, Romanova, Lovelace, Kulkarni, Long, Lii et al, Zanni & Ferreira, .......

Romanova et al. 12

Zanni & Ferreira '13
Outstanding Issues:
(uncertainties, possible applications…)

- Magnetosphere boundary vs disk inner radius
- Magnetic linkage between star and disk
  quasi-cyclic behavior
- Magnetosphere outflows
- Torque on the star: Spinup/spindown
- Misaligned dipole: Effect on disks
- Spin-disk misalignment: Application to exoplanets
Dipole Field Invaded by a Conducting Disk

Disk is a good conductor:
Diffusion of $B$ into disk:

$$t_{\text{diff}} \sim \frac{H^2}{\eta} \sim \frac{1}{\alpha \Omega}$$

$$\eta \sim \nu = \alpha H c_s \quad \text{(MRI)}$$
Dipole Field Invaded by a Conducting Disk

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Instabilities at inner edge (KH, RT, reconnection)

\[ \Rightarrow \text{Boundary layer} \]
Magnetosphere Boundary Layer

\[ r_m \rightarrow r_m + \Delta r_m \]
Transition from \( \Omega_K \) to \( \Omega_* \)
\[ B_{\phi+} = -\zeta B_z \]

Magnetic torque on BL (per unit area)
\[
\frac{r B_z B_{\phi+}}{2\pi} = \sum \frac{d(r^2\Omega)}{dt} = \Sigma v_r \frac{\partial(r^2\Omega)}{\partial r}
\]

\[ r_m \sim \left( \frac{\mu^4}{GM\dot{M}^2} \right)^{1/7} \]
depends on \( \zeta, \Delta r_m \).
Star-Disk Linkage
(Width, Time-dependence…)

Linked fields are twisted by differential rotation…
→ Field inflates, breaks the linkage

Aly; Lovelace et al.; Uzdensky,…
Star-Disk Linkage
(Width, Time-dependence…)

Linked fields are twisted by differential rotation…
⇒ Field inflates, breaks the linkage

Maximum twist: \[
\left| \frac{B_{\phi}^+}{B_z} \right|_{\text{max}} \sim 1
\]

Aly; Lovelace et al.; Uzdensky,…
Star-Disk Linkage: Quasi-cyclic behavior
(Width, Time-dependence…)

Stellar field penetrates the inner region of disk;
Field lines linking star and disk are twisted $\rightarrow$ toroidal field $\rightarrow$ field inflation
Reconnection of inflated fields restore linkage

Inevitable…
Star-Disk Linkage: Quasi-cyclic behavior
(Width, Time-dependence…)

Stellar field penetrates the inner region of disk;
Field lines linking star and disk are twisted --> toroidal field --> field inflation
Reconnection of inflated fields restore linkage

Application: Connection with QPOs in LMXBs (and other systems)?
Quasi-Periodic Oscillations (QPOs)

Power density spectrum of x-ray flux variations of accreting millisecond pulsars

Van der Klis 2005
Star-Disk Linkage: Quasi-cyclic behavior
(Width, Time-dependence…)

Stellar field penetrates the inner region of disk;
Field lines linking star and disk are twisted --> toroidal field --> field inflation
Reconnection of inflated fields restore linkage

Application: Episodic outflow … Connection with observations?
Ejection from Magnetospheric Boundary

Romanova et al. 2009

Zanni & Ferreira 2013
Star-Disk Linkage: Quasi-cyclic behavior
(Width, Time-dependence…)

Stellar field penetrates the inner region of disk;
Field lines linking star and disk are twisted --> toroidal field --> field inflation
Reconnection of inflated fields restore linkage

QUESTION: On average, what is the width of the linked region? $\Delta r$
Torque on Star

\[
\frac{dJ_x}{dt} \approx \dot{M}_{\text{acc}} r_m^2 \Omega(r_m) + T_m
\]

\[
T_m \approx - (r^2 B_z B_{\phi+})_{r_m} \Delta r = \zeta (r^2 B_z^2)_{r_m} \Delta r \quad \text{for } B_{\phi+} = -\zeta B_z
\]

Note: \(|\zeta| \sim 1: \zeta > 0 \text{ when } r < r_c \text{ and } \zeta < 0 \text{ when } r > r_c\)
Torque on Star

\[ \frac{dJ_*}{dt} \approx \dot{M}_{\text{acc}} r_m^2 \Omega(r_m) + T_m \]

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**Note:** \(|\zeta| \sim 1\): \(\zeta > 0\) when \(r < r_c\) and \(\zeta < 0\) when \(r > r_c\)

**Issues:**
Equilibrium spin: Protostars, millisecond pulsars, long-period pulsars
Application: Rotation of Protostars: why 10% of breakup?

Gallet & Bouvier 2013
Torque on Star

\[
\frac{dJ_x}{dt} \approx \dot{M}_{\text{acc}} r_m^2 \Omega(r_m) + T_m
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\]

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Issues:
Can we understand spinup/spindown of X-ray pulsars?
Spinup/Spindown of Accreting X-ray pulsars

Bildsten et al. 1997
4U1626-67
7.66s
Transition lasted 150 days

Camero-Arranz et al. 2010, 2012
Misaligned Dipole

Accretion through instabilities

Funnel flow to polar caps
Misaligned Dipole: Exciting Waves in Disks

Vertical force on disk:

\[ F_z(r, \varphi, t) = F_\omega(r) \exp(i m \varphi - i \omega t) \]

\[ m = 1, \quad \omega = \Omega_*, 2\Omega_* \]

→ Excitation of Bending waves in the disk
Misaligned Dipole: Exciting Waves in Disks

Vertical force on disk:

\[ F_z(r, \varphi, t) = F_\omega(r) \exp(i m \varphi - i \omega t) \]

\[ m = 1, \quad \omega = \Omega_*, 2\Omega_* \]

\[ \Rightarrow \text{Excitation of Bending waves in the disk} \]

Perturbations most “visible” at Lindblad/Vertical Resonance

\[ \omega - \Omega = \Omega_\perp \sim \Omega \]

\[ \Rightarrow \Omega(r_L) = \frac{\omega}{2} = \frac{\Omega_*}{2}, \Omega_* \]

**Question:** QPOs….
kHz QPOs in Accreting Millisecond Pulsars

\[ \nu_s = 401 \text{ Hz}, \quad \nu_h - \nu_l \simeq \frac{\nu_s}{2} \ (\pm \text{a few Hz}) \]

XTE J1807.4-294: \[ \nu_s = 191 \text{ Hz}, \quad \nu_h - \nu_l \simeq \nu_s \]

Beating of high-freq. QPO with perturbed fluid at L/VR ?

Van der Klis 2005
Stellar Spin – Disk Misalignment

Standard story: S//L
There is a magnetic torque which tends to make the inner disk warp on timescale $>>$ dynamical time (rotation/orbital period)
A Laboratory Experiment
A Laboratory Experiment

DL, Foucart & Lin 2011
Magnetically Threaded disk:

Torque on disk (per unit area):
\[ \mathbf{N} \propto -\hat{\mathbf{l}} \times (\hat{\mathbf{\mu}} \times \hat{\mathbf{l}}) \]

Averaging over stellar rotation:
\[ \mathbf{N} \propto -\hat{\mathbf{l}} \times (\hat{\mathbf{\omega}}_s \times \hat{\mathbf{l}}) \]

Warping torque
Possible Connection to (Exo)Planetary Systems
Many “hot Jupiters” have Misaligned $S^*_L - L_p$
S*-L_p misalignment in Exoplanetary Systems

The Importance of few-body interactions

1. Kozai + Tide migration by a distant companion star/planet
   (e.g., Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al. 12, Katz et al. 12)

2. Planet-planet Interactions
   -- Strong scatterings
   (e.g., Rasio & Ford 96; Chatterjee et al. 08; Juric & Tremaine 08)
   -- Secular interactions ("Internal Kozai", chaos)
   (e.g, Nagasawa et al. 08; Wu & Lithwick 11)

-- Chaotic stellar spin evolution during Kozai
   (Storch, Anderson & DL 2014)
Is “High-e Migration” the whole story for producing hot Jupiters and S-L misalignments?

**Likely NO.**

-- Companion? Initial conditions? (e.g., Knutson et al. 2014)

-- Can produce distribution of period, ecc, misalignment? (Naoz+12, Petrovich+14)

-- Paucity of high-e proto-hot Jupiters (Socrates et al. 2012; Dawson et al. 2012)

-- Stellar metallicity trend of hot Jupiters ➔ Two mechanisms of migrations (Dawson & Murray-Clay 2013)

-- Misaligned multiplanet systems:
   - Kepler-55 (2 planets 10.5 & 21 days ➔ 40-55 deg from seismology; Huber et al 2013)
   - Kepler-9 (3-planets; Walkowicz & Basri 2013)?
   - Other Candidates: Hirano et al. 2014
Alternative: Primordial Misalignments
between Stellar Spin and Protoplanetary Disk


-- Perturbation of Binary on Disk (Batygin 2012; Batygin & Adams 2013; Lai 2014)
Recall: Magnetic torques from the star want to make the inner disk warp and precess...

But disk will want to resist it by internal stresses (viscosity or bending wave propagation)

\[
\frac{\partial}{\partial t} \left( \Sigma r^2 \Omega \hat{l} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \Sigma V_R r^3 \Omega \hat{l} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( Q_1 I r^2 \Omega^2 \hat{l} \right) \\
+ \frac{1}{r} \frac{\partial}{\partial r} \left( Q_2 I r^3 \Omega^2 \frac{\partial \hat{l}}{\partial r} + Q_3 I r^3 \Omega^2 \hat{l} \times \frac{\partial \hat{l}}{\partial r} \right) + N_m
\]
Back-reaction Torque on Star: What is happening to the stellar spin direction? (Is there secular change to the spin direction?)

- Warping torque

- Back-reaction torque

→ Stellar spin may be misaligned with disk axis
Accretion tends to align S & L:

Accretion torque \( N_{\text{acc}} \sim \dot{M} \sqrt{GM_* r_{\text{in}}} \)

Magnetic misalignment torque: \( N_{\text{mag}} \sim \mu^2 / r_{\text{in}}^3 \)

For \( r_{\text{in}} \sim \left( \frac{\mu^4}{GM_* \dot{M}^2} \right)^{1/7} \)

\( \Rightarrow \quad N_{\text{acc}} \sim N_{\text{mag}} \)
Evolution of the stellar spin

\[ \frac{d}{dt} (J_s \dot{\omega}_s) = \mathcal{N} = \mathcal{N}_{\text{acc}} + \mathcal{N}_m + \mathcal{N}_{\text{sd}} \]

\[ \mathcal{N}_{\text{acc}} = \lambda \dot{M} \sqrt{GMr_{\text{in}}} \dot{\ell}_{\text{in}}, \quad \lambda \sim 1 \text{ (or less)} \]

\[ \mathcal{N}_m = \text{backreaction of magnetic (warping & precessional) torques} \]

\[ \mathcal{N}_{\text{sd}} = -|\mathcal{N}_{\text{sd}}| \dot{\omega}_s \]

(Each term is of order \( \mathcal{N}_0 = \dot{M} \sqrt{GMr_{\text{in}}} \))

\[ \frac{d \cos \theta_{\text{sd}}}{dt} = \frac{\mathcal{N}_0}{J_s} \sin^2 \theta_{\text{sd}} \left( \lambda - \bar{\zeta} \cos^2 \theta_{\text{sd}} \right) \]

\[ \bar{\zeta} = \frac{\zeta \cos^2 \theta_*}{6\eta^{7/2}} \sim 1 \]

Spin evolution timescale:

\[ t_{\text{spin}} = (1.25 \text{ Myr}) \left( \frac{M_*}{1 \text{ M}_\odot} \right) \left( \frac{\dot{M}}{10^{-8} \text{ M}_\odot \text{ yr}^{-1}} \right)^{-1} \left( \frac{r_{\text{in}}}{4R_*} \right)^{-2} \frac{\omega_s}{\Omega(r_{\text{in}})} \]
Evolution of the stellar spin

\[
\frac{d \cos \theta_{sd}}{dt} = \frac{N_0}{J_s} \sin^2 \theta_{sd} \left( \lambda - \bar{\zeta} \cos^2 \theta_{sd} \right)
\]

\[
\bar{\zeta} = \frac{\zeta \cos^2 \theta_*}{6\eta^{7/2}} \quad (\sim 1)
\]

\[\lambda > \bar{\zeta}\]

“weak” warping torque

\[\lambda < \bar{\zeta}\]

“strong” warping torque
For Isolated star-disk systems:

Magnetic torque tends to produce spin-disk misalignment, But competes with accretion

→ May or may not produce small/modest misalignment
  (e.g., Solar system 7 degree?)
Star-Disk-Binary Interactions

Gravitational interactions…

Now include Accretion and Magnetic Torques
No accretion/magnetic

Accretion/magnetic damps SL-angle

Accretion/magnetic increases SL-angle

DL2014
Summary

Magnetic star - disk interaction

**Rich MHD and plasma physics:** connection to other field
- Interchange instabilities, reconnection, field winding/inflation,
- outflows, waves, resonances…

**Wide Applications** (accreting NSs, WDs, protostars):
- Variabilities (QPOs), jets, spin equilibrium, spinup/down,
- warp/prcession, exoplanets, protoplanetary disks
THANKS